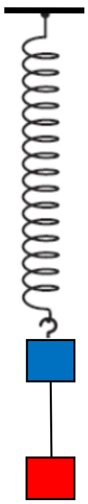
Teacher notes
Topic C

Simple harmonic oscillations

The spring has spring constant k = 162 N m^{-1}. The string is cut. Write down the equation giving the displacement of the 0.50 kg mass as a function of time.



Before the string is cut the tension in the spring is 8.0 N and so the extension is $x = \frac{8.0}{162} = 4.94\text{ cm}$. The new equilibrium position has the spring extended by $x = \frac{5.0}{162} = 3.09\text{ cm}$. The displacement at t = 0 when the string is cut is then $x = 4.94 - 3.09 = 1.85\text{ cm}$. This will be the amplitude of oscillations x_0.

$$x = x_0 \sin(\omega t + \phi)$$
$$v = \omega x_0 \cos(\omega t + \phi)$$
$$a = -\omega^2 x_0 \sin(\omega t + \phi)$$

At t = 0, x = –1.85 cm: this means that $-1.85 = 1.85\sin(0 + \phi)$. I.e. $\sin\phi = -1 \Rightarrow \phi = \frac{3\pi}{2}$. The angular frequency is given by $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{162}{0.50}} = 18\text{ s}^{-1}$. So we expect $x = 1.85\sin(18t + \frac{3\pi}{2})$. We can check f this makes sense. The velocity at $t = 0$ is $v = \omega x_0 \cos(0 + \frac{3\pi}{2}) = 0$ as it should be. The acceleration at $t = 0$ is $a = -\omega^2 x_0 \sin(0 + \frac{3\pi}{2}) = +\omega^2 x_0 = 18^2 \times 1.85 \times 10^{-2} = 6.0\text{ m s}^{-2}$. This is as it should be because right after

after the string is cut the tension in the spring is still 8.0 N and so the net force on the mass is 8.0 – 5.0 = 3.0 N. The initial acceleration is then $a = \frac{3.0}{0.50} = 6.0\text{ m s}^{-2}$.

Hence the answer to the problem is $x = 1.85\sin(18t + \frac{3\pi}{2})$.